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Heat transfer correlations by symbolic regression

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Abstract

We describe a methodology that uses symbolic regression to extract correlations from heat transfer measurements by searching for both the form of the correlation equation and the constants in it that enable the closest fit to experimental data. For this purpose we use genetic programming modified by a penalty procedure to prevent large correlation functions. The advantage of using this technique is that no initial assumption on the form of the correlation is needed. The procedure is tested using two sets of published experimental data, one for a compact heat exchanger and the other for liquid flow in a circular pipe. In both situations, predictive errors from correlations found from symbolic regression are smaller than their published counterparts. A parametric analysis of the penalty function is also carried out.

Keywords: Heat transfer; Correlations; Symbolic regression; Genetic programming; Heat exchanger

1. Introduction

In the design, selection and control of thermal components for industrial and commercial applications, it is necessary to predict their performance under specific conditions of operation. Though in theory this calculation can be carried out from first principles by formulating the governing equations, complexities arising from factors like turbulence, temperature dependence of properties, and the geometry makes it difficult to achieve in practice. As a result, most calculations are based on experimental data. The information is compressed in the form of correlations from which the heat transfer coefficient can be obtained. Most commonly, correlations are developed in terms of dimensionless groups like the Nusselt, Reynolds and Prandtl numbers; sometimes for greater generality geometrical factors are also included. Assuming a functional relationship between the groups with a certain number of free constants, a regression analysis to minimize the error between predicted and experimental values is carried out to determine the appropriate values of the constants.

A disadvantage of this procedure is that predictive errors in the heat transfer rate are normally larger than the experimental uncertainties from which the correlation was generated. Assumptions such as using average transfer coefficients or constant property values [1] and the fact that the error minimization function may have more than one local minimum [2,3] are among the reasons for this loss of accuracy. Another source of error is the specific form of the correlation function assumed for the regression analysis. The functional form is selected on the basis of simplicity, compactness and common usage [4], but cannot be completely justified from first principles. There is usually not much physics behind the choice of the form. Although power laws are commonly used in heat transfer studies, a variety of other forms have also been used [5], though it is not obvious how the form should be chosen. As an example, for heat exchangers Pacheco-Vega et al. [6] have shown that *different* functional forms may predict performance with more or less similar accuracy. It would thus be

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Nomenclature

A_r	area ratio	Pr	Prandtl number
a_1, a_2	penalty parameters	p_c	probability of crossover
$\mathbf{C}$	set of constants	p_m	probability of mutation
D	inner diameter of pipe	Q_f, Q_j, Q_{Nu}	penalized fitness function
$\mathbf{F}$	set of operators	Re	Reynolds number
f	correlation function	S_j, S_{Nu}	variance of error
F_f, F_j, F_{Nu}	fitness function	$\mathbf{T}$	terminal set
g	penalty function	x_j	variable
G	generation number		
G_{max}	maximum number of generations	*Greek symbol*	
j	Colburn j-factor	μ	dynamic viscosity of fluid
L	length of pipe		
L_f, L_j, L_{Nu}	size of correlation	*Subscripts and superscripts*	
M	population size	e	experimental value
N	number of experimental data sets	p	predicted value
N_v	number of variables	t	target value
Nu	Nusselt number	w	wall

advantageous to have an algorithmic way to determine the best correlation that fits experimental data without the need to assume its functional form.

The genetic algorithm (GA) [7,8] is an optimization technique based on stochastic, evolutionary principles that is used to find global extrema of a given function. Genetic programming (GP) [9] is a symbolic regression extension that works with a set of possible functions to find the best for a given set of data. Applications of GP to thermal engineering are scarce: the correlations obtained by Lee et al. [10] for critical heat flux for water flow in vertical round pipes and Pacheco-Vega et al. [11] for artificial heat-exchanger data are among the very few.

The aim of the present study is to describe a methodology based on GP to develop heat transfer correlations that can be used to predict the performance of thermal components. Since compact forms of the correlations are to be preferred, the standard procedure will be modified by a penalty function that weights against complicated forms. The procedure is described first. Then, two sets of published experimental data, one corresponding to heat transfer in compact heat exchangers and the other to heating and cooling of liquids in pipes, are used to demonstrate the capability of GP to find accurate correlations. The effect of the parameters of the penalty function on the results is also analyzed.

2. Genetic programming

2.1. Description

GP is a soft computing search technique in which computer codes, representing functions as parse trees, evolve as the search proceeds. The objective is to extremize a certain quantity called the *fitness function*. Developed originally to automatically generate computer programs, it has been used in a variety of applications, e.g., finance [12], electronic design [13], signal processing [14], and system identification [15], among others. GP is discussed in detail in the monograph by Koza [9].

Compared to the GA [7,8], in GP functions take the place of numbers in an attempt to find the best solution to a particular problem by genetically recombining a population of individuals that portray candidate solutions. This is achieved by using tree-structured representations of functions; an example of the function $5x\cos(5x+1)$ is shown in Fig. 1(a). Branch nodes may be operators with one or two arguments (such as sin, cos, exp, log, +, −, ∗, /, ∧), or may be Boolean (such as AND, OR, NOT) or conditional (IF–THEN–ELSE, etc.) operators. Leaf or terminal nodes, on the other hand, are the variables ($x_j, j = 1,\ldots,N_v$) in a particular problem, or constants to be

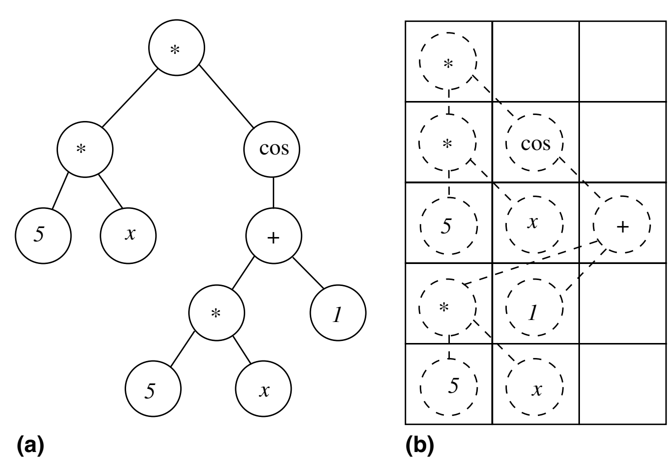

Fig. 1. Representation of function $5x\cos(5x+1)$ as (a) parse tree, and (b) array.

determined. For our purposes we will use the set of operators

$$\mathbf{F} = \{+, -, *, /, \wedge\} \tag{1}$$

for the branch nodes, where numerical values are returned on application of each operator. The division operator, /, is protected such that it is prevented from being singular if there is a zero in the denominator [9]. The set of terminals will be

$$\mathbf{T} = \{x_j, \mathbf{C}\}, \tag{2}$$

where x_j are the variables given as data, while $\mathbf{C} \in \mathbb{R}$ is a set of constants which have to be determined as part of the solution.

2.2. Computer representation

In coding the algorithm, the representation of functional forms that maintains a correct syntax depends on the programming language being used. Due to the natural way of portraying these tree structures, GP was ordinarily coded in LISP [9,16]. However, other object-oriented programming languages like C or C++ have also been used with an increased speed in the computations. We have coded our algorithm in MATLAB which allows handling different data-types in a straight-forward manner. Trees representing the correlations are stored as rectangular arrays. We define the "size," L_f, of the correlation to be the number of rows multiplied by the number of columns of the smallest rectangular array representation. The array representation of the function $5x\cos(5x+1)$ is shown in Fig. 1(b) with $L_f = 5 \times 3 = 15$.

2.3. Fitness and penalty

Since the objective is to minimize the variance of the error between predictions and the data, it is natural to define the fitness as the reciprocal of the variance so that

$$F_f = \left(\frac{1}{N} \sum_{i=1}^{N} [f^{\mathrm{t}}(x_j)_i - f^{\mathrm{p}}(x_j, \mathbf{C})_i]^2 \right)^{-1}, \tag{3}$$

where $f^{\mathrm{t}}(x_j)$ for $i = 1, \ldots, N$, are target data, and $f^{\mathrm{p}}(x_j)$ for $i = 1, \ldots, N$, are the predicted values from candidate correlations. In heat transfer applications, f is either transfer conductances or a dimensionless form of the heat transfer coefficients; x_j for $j = 1, \ldots, N_{\mathrm{v}}$, are dimensionless groups such as the Reynolds number, Prandtl number, and geometrical parameters.

One problem with using the fitness directly as defined in Eq. (3) is that it may result in correlations of complicated forms with many terms. To prevent large correlation functions and favor more compact ones, the fitness function can be penalized according to the size of each correlation. Though this can be done in many ways, we follow McKay et al. [17] and define a penalized fitness as

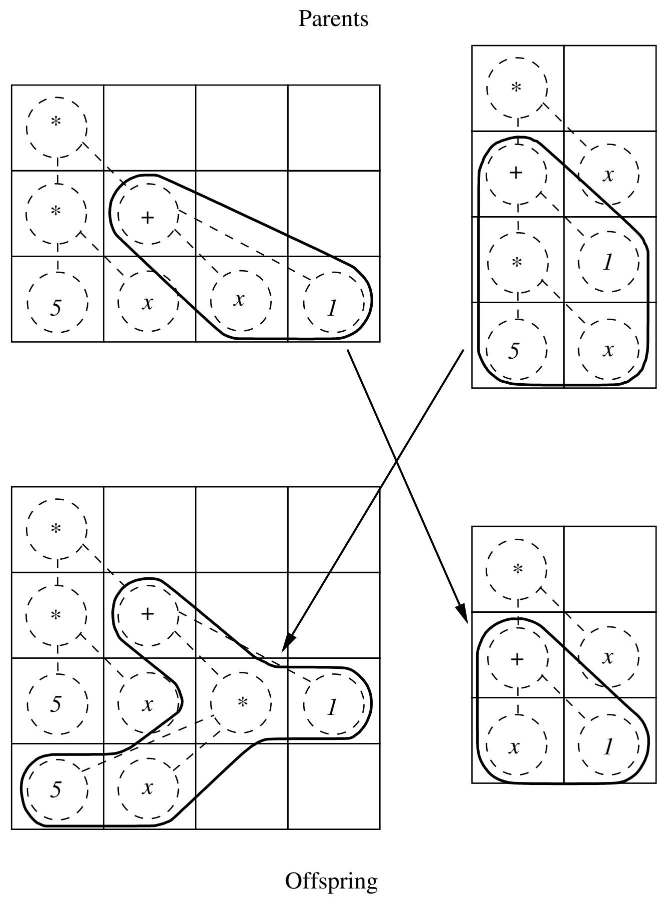

Fig. 2. Crossover: parents are $5x(x+1)$ and $x(5x+1)$, and offspring are $5x(5x+1)$ and $x(x+1)$.

$$Q_f = F_f g(L_f), \tag{4}$$

where

$$g(L_f) = \frac{1}{1 + \exp[a_1(L_f - a_2)]} \tag{5}$$

is a sigmoidal penalty function with prescribed a_1 and a_2. In Section 4 we will analyze the effect that a_1 and a_2 have on the results.

2.4. Crossover and mutation

In crossover two parents interchange parts of their trees to produce two offspring following a process of cutting and grafting. The crossover points may be different in each parent, as illustrated in Fig. 2. Taking the set of elements located "below" a chosen operator and the operator itself from each parent, crossover can be achieved by grafting these into the other parent at the appropriate location. Mutation is applied on a node-by-node basis by random alteration of a branch or terminal node as illustrated in Fig. 3. Note that when applying these procedures, one must make sure that the resulting functions are syntactically acceptable.

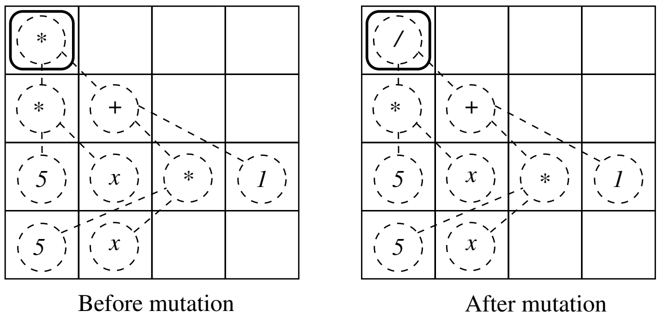

Fig. 3. Mutation: before mutation function is $5x(5x+1)$, and after mutation is $5x/(5x+1)$.

2.5. Procedure

The steps are the following.

- *Creation of population*: For the first generation a set of M correlations coded as parse trees is randomly generated from **F** and **T**. For the following generations, the old population is used.
- *Evaluation of fitness*: For each member of the population, the value of the fitness is calculated.
- *Selection for reproduction*: The probability distribution for the next generation, on the basis of which parents are selected for replacement, is calculated from the fitness values. A number of selection strategies exist in the literature, e.g., fitness proportionate or elitism [9], among others. Here we use the so-called tournament method since it guarantees diversity in the population [18]. The method uses two parameters, a so-called population gap representing the percentage of individuals with better fitness, and a tournament gap which provides the number of individuals randomly selected from the population for reproduction.
- *Application of genetic operators:* GP guides the search by applying the genetic operators *crossover* and *mutation* to parents selected on the basis of their fitness function. Once the parents are selected, crossover and mutation are applied according to preselected probabilities p_c and p_m, respectively.
- *Determination of constants*: During the search, the optimum form of the correlation found at any iteration of the algorithm may not have optimal values of constants **C**. This may cause the search path to deviate from the optimum as the search proceeds, and it may prevent the convergence to the best possible correlation relative to both functional form and the constants. Thus it is necessary to complement the GP with an optimization of the set of constants **C**. In the present work, this is done using the GA and supplemented by local optimization using the Nelder–Mead algorithm [19]. These are applied periodically after a number of generations.
- *Creation of new population*: Once crossover and mutation have been applied to the complete population, a new population that keeps the fittest member of the previous generation is created.

The process is repeated until some criterion based on convergence or maximum number of generations, G_{max}, is achieved. Since this is a probabilistic technique, every run gives a slightly different answer. To understand this inherent variation, it is useful to perform multiple runs along with a statistical analysis of the results.

3. Compact heat exchanger data correlation

The procedure is now applied to data obtained from experimental measurements. Heat exchangers are a common example of thermal components, and empirical correlations have been proposed by Abu Madi et al. [20], Kim et al. [21] and Wang et al. [22] for single-phase flow conditions, and McQuiston [23] and Khartabil [24] for condensing conditions.

We consider experimental data that were obtained and reported by McQuiston [25] from a series of tests on a fin-tube compact heat exchanger. This was a multi-row multi-column heat exchanger with nominal size of 127 mm × 305 mm in which air was used as the over-tube and water as the in-tube fluid. The focus of the study was the air-side heat transfer which was reported in terms of Colburn j-factors. High Reynolds-number turbulent flow in the water side was used to yield the thermal resistance of the air side only. Though the measurements covered a wide range of operating conditions, i.e. dry surface, drop-wise and film condensation, only the dry-surface data will be considered here.

From measurement data, the correlation proposed by McQuiston [23] is

$$j = 0.0014 + 0.2618 Re^{-0.4} A_r^{-0.15}, \tag{6}$$

where j is the Colburn j-factor, Re is Reynolds number, and A_r is a non-dimensional geometrical parameter representing an air-side area ratio.

To perform symbolic regression for these data, the fitness function is defined as

$$F_j = (S_j)^{-1} = \left(\frac{1}{N}\sum_{i=1}^{N}(j_i^e - j_i^p)^2\right)^{-1}, \tag{7}$$

where j_i^e, $i = 1,\ldots,N$, are the measurements and j_i^p, $i = 1,\ldots,N$, are the predicted values from each of the M correlations in the population. Here we seek a correlation function and the corresponding constants that maximize F_j (or minimize S_j). We choose: $M = 100$, $G_{max} = 800$, $p_c = 0.8$, $p_m = 0.2$, $a_1 = 0.2$ and $a_2 = 30$. The terminal sets include the variables $x_1 = Re$, and $x_2 = A_r$. The population and tournament gaps are 10% and 2, respectively. The determination of constants is done every 10 generations, and the procedure repeated 10 times.

Fig. 4 illustrates a typical evolution of the algorithm with respect to generation number G. The two curves in the figure correspond to the values of the unpenalized and penalized fitnesses, F_j and Q_j respectively, from the best correlation in each generation. After 400 generations

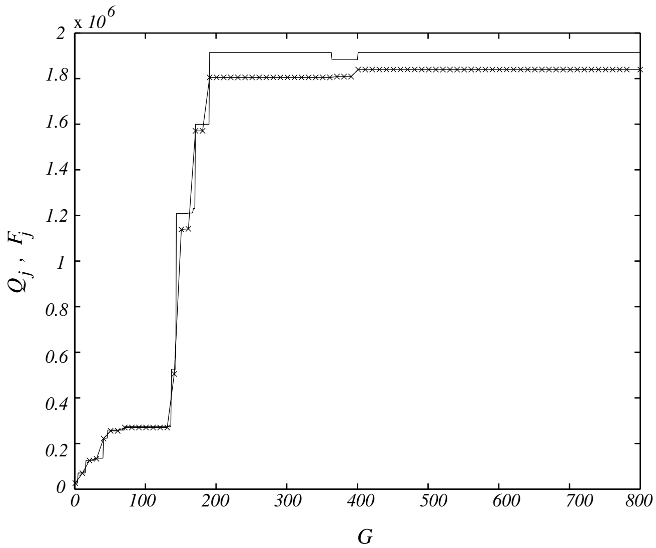

Fig. 4. Evolution of fitness and penalized fitness with generation number. — fitness function F_j, -×- penalized fitness Q_j.

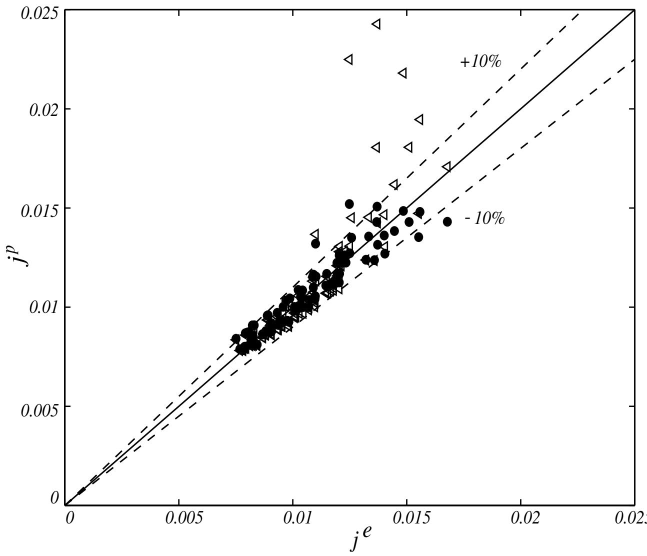

Fig. 5. Experimental vs. predicted j-factors for compact heat exchanger from Eq. (10): (●). Also shown are predictions of Eq. (6) [23]: (◁). Straight line is the perfect prediction.

it is observed that both curves level off with values of F_j being larger than those of Q_j. This is to be expected since Q_j rejects good candidate correlations that are large in size. A consequence of this is that though both curves follow similar paths, at about $G = 350$ the F_j-curve decreases while Q_j keeps increasing. Since the algorithm keeps the best correlation from the previous generation based on Q_j, rather than F_j, a correlation that has a large F_j but is also large in size, may not be preserved by the algorithm.

The following are two examples of the correlations that result from the algorithm:

$$j = \frac{1.82}{103.81 + 0.0299Re + A_r}, \tag{8}$$

$$j = \frac{66.39 - 0.4456A_r}{3881.61 + Re}. \tag{9}$$

Though these correlations are different in form, and their sizes, L_j, are 10 and 16 respectively, their rms errors in predictions of the j-factor are close. This multiplicity of solutions in functional space was also noticed by Pacheco-Vega et al. [11] using artificial data. After the procedure was run a number of times, a correlation with a slightly more complex form and larger size, $L_j = 24$, but better prediction is found to be

$$j = \frac{2205.32}{1.39 \times 10^5 + 24.16Re + A_rRe}. \tag{10}$$

Table 1
Comparison of RMS errors for heat exchanger

Prediction method	Error (%)
McQuiston, Eq. (6) [23]	14.74
Eq. (8)	6.32
Eq. (9)	6.24
Pacheco-Vega et al. [3]	6.21
Eq. (10)	6.18

Table 1 shows a comparison of the rms percentage error in j obtained from the GP-based correlations above with the published counterparts. The errors indicated are in descending order of magnitude. It is observed that, regardless of differences in accuracy, all the correlations found by this method give a smaller error than that of Eq. (6). Also shown in the table are the results of the correlation developed by Pacheco-Vega et al. [3] using global regression with the same functional form as given by Eq. (6). Though this global-regression-based correlation is the best possible that can be obtained from the assumed functional form, Eq. (10) is seen to give a slightly smaller error.

A comparison between the experimentally determined j-factor and that predicted from Eq. (10) is illustrated in Fig. 5. The predictions from Eq. (6) are also included as a reference. The scatter in the predictions from the GP-based correlation is much smaller.

4. Effect of penalty parameters

Though different penalty functions may be used to limit the size of the correlations, one of the advantages of the sigmoidal form in Eq. (5) is that, since it is bounded and its denominator is non-zero, it prevents the fitness in Eq. (4) from becoming either unbounded or singular and thus avoids computational problems. However, the choice of a_1 and a_2 may affect the results. With the other parameters fixed to the values used before, we take the data set of McQuiston [25] and vary a_1 and a_2 to analyze their effect on the results. Three runs were made for each a_1 and a_2 value.

Fig. 6 shows the results when a_1 is held constant and a_2 is varied. Fig. 6(a) shows the penalty function vs. correlation size with the result of each run also marked on it, and Fig. 6(b) the fitness values of the results. a_2 is a

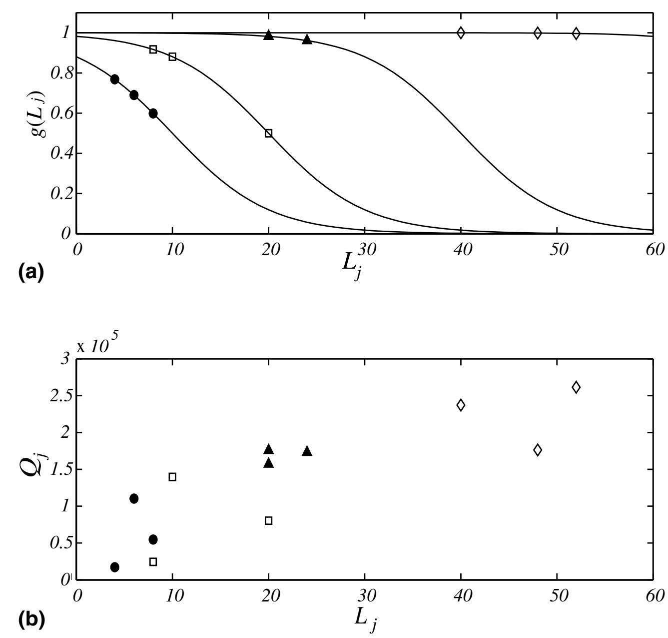

Fig. 6. (a) Penalty and (b) penalized fitness functions vs. correlation size for different a_2: (●) 10, (□) 20, (▲) 40, (◇) 80.

nominal size of the correlation that simply shifts the penalty function horizontally. A larger a_2 enables a larger correlation size with a smaller error to be selected. The choice of a_2 is subjective in that a smaller error is to be preferred, but at the expense of functional complexity.

Fig. 7 is for variable a_1 and constant a_2, with Fig. 7(a) showing the penalty function and Fig. 7(b) the penalized fitness function. It can be seen that a_1 is a measure of the slope of the penalty function and determines how strongly

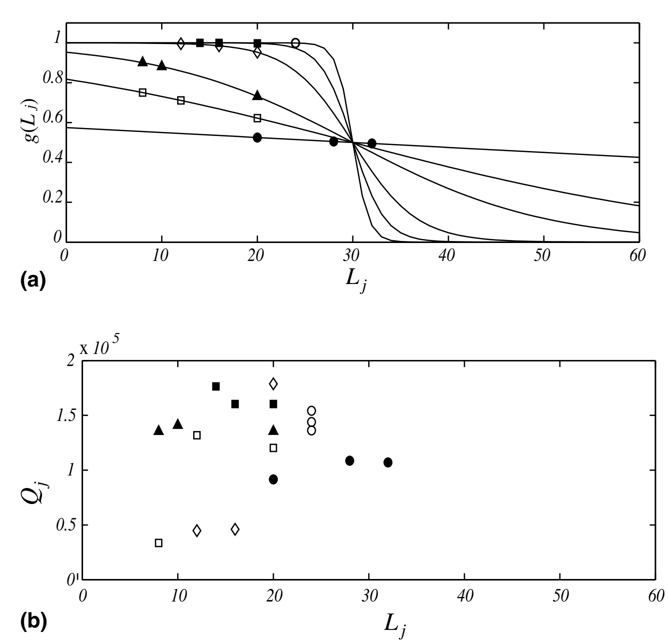

Fig. 7. (a) Penalty and (b) penalized fitness functions vs. correlation size for different a_1: (●) 0.01, (□) 0.05, (▲) 0.1, (◇) 0.3, (■) 0.6, (○) 1.2.

a large functional form of the correlation is penalized. For small values of a_1, the penalty function changes smoothly providing a gradually increasing rejection of large correlations while for the large values the fitness curve shows a jump, i.e. a sudden onset of rejection, which is not very conducive to a smooth variation in correlation size during the search procedure. Though the choice of a_1 is subjective, an intermediate a_1 is appropriate for finding relatively compact well-fitted correlations. By looking at its limiting values we can see that choosing a large a_1 would be equivalent to picking a maximum correlation size for which there is no need of a penalty function. If, on the other hand, a_1 is small, then it is also equivalent to not choosing a penalty function.

5. Pipe-flow data correlation

Experimental data for heating and cooling of liquids in pipes were reported by Sieder and Tate [26]. These data and the corresponding correlation are frequently used to calculate heat transfer coefficients in laminar flow in pipes during design calculations. Using three distinct oils as working fluids, a total of 67 experimental runs were reported. The experimental results included the Nusselt Nu, Reynolds Re, and Prandtl Pr numbers, as well as the viscosity ratio μ/μ_w. Here μ and μ_w are the fluid dynamic viscosities calculated at the average and wall temperatures, respectively. Also given were the length, L, and the inner pipe diameter, D, of the concentric-tube heat exchanger where the experiments were performed.

On assuming an exponent of 1/3 for Re, Pr and D/L, the correlation developed graphically by Sieder and Tate [26] is

$$Nu = 1.86 Re^{1/3} Pr^{1/3} \left(\frac{\mu}{\mu_w}\right)^{0.14} \left(\frac{D}{L}\right)^{1/3}, \tag{11}$$

with a corresponding range of applicability. From the same data, using the functional form of Eq. (11) and the same exponent for D/L, a different correlation was produced numerically by Levenspiel et al. [27]. Their correlation is

$$Nu = 4.22 Re^{0.288} Pr^{0.243} \left(\frac{\mu}{\mu_w}\right)^{0.142} \left(\frac{D}{L}\right)^{1/3}. \tag{12}$$

We apply the present algorithm to the data set of Sieder and Tate [26] to find the best fit correlation. The parameters chosen for the procedure are: $M = 200$, $G_{max} = 800$, $p_c = 0.8$, $p_m = 0.2$, $a_1 = 0.05$ and $a_2 = 50$. The values for the population and tournament gaps are 10% and 2, respectively. The variables are $x_1 = Re$, $x_2 = Pr$, $x_3 = \mu/\mu_w$, and $x_4 = D/L$. Determination of constants is performed every 10 generations.

For each of the M correlations in the population, the error in the Nusselt number between predictions and experiments is calculated from

$$S_{Nu} = \frac{1}{N} \sum_{i=1}^{N} \left(\frac{Nu_i^e - Nu_i^p}{Nu_i^e}\right)^2, \tag{13}$$

Table 2
Comparison of RMS errors for liquids flow in pipes

Prediction method	Error (%)
Sieder and Tate [26]	13.79
Levenspiel et al. [27]	11.19
Eq. (14)	10.69

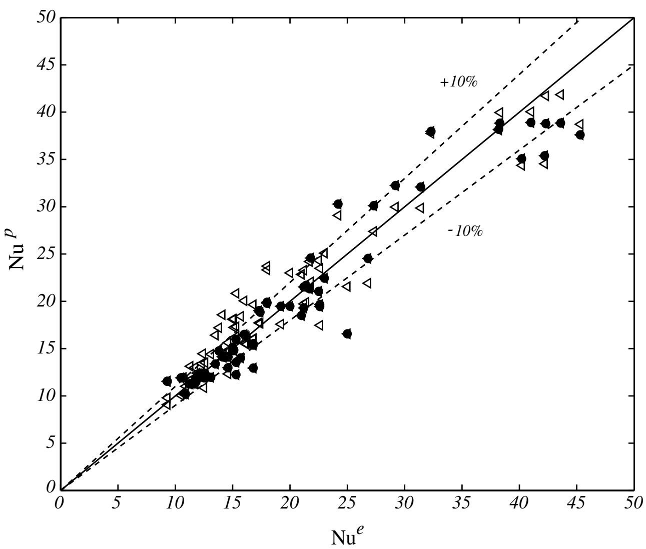

Fig. 8. Experimental vs. predicted Nusselt numbers for liquids inside pipes from Eq. (14): (●). Also shown are predictions of Eq. (11) [26]: (◁). Straight line is the perfect prediction.

where Nu_i^e are the experimental, and Nu_i^p the predicted values, for $i = 1, \ldots, N$. The fitness function is thus defined as $F_{Nu} = (S_{Nu})^{-1}$, and the penalized fitness as $Q_{Nu} = (S_{Nu})^{-1} g(L_{Nu})$, where g is given in Eq. (5). The procedure was carried out 10 times, with the most frequent functional form found being

$$Nu = 11.28 + \frac{3.81Re[1 + 8.49(\mu/\mu_w)]}{Re(\mu/\mu_w) - 0.1907Pr + 12.89(D/L)}. \qquad (14)$$

The predictions of Nu from Eq. (14), along with the published correlations previously discussed, are summarized in Table 2. The percentage error from the GP-based correlation is better than those given by Eqs. (11) and (12).

Fig. 8 is a graphical comparison between the predictions from Eqs. (14) and (11). The accuracy in the predictions from the GP-based correlation is in general better, mainly in the low-value range of the Nusselt number. The price paid for this accuracy, however, is the slightly increased functional complexity; its size is $L_{Nu} = 48$ as compared to $L_{Nu} = 40$ of Eqs. (11) and (12).

6. Conclusions

Correlations obtained from experimental data are commonly used in the estimations of the heat rate in thermal components. Most often this reduction of experimental data to correlations is based on first choosing a specific functional form of the correlation for which the constants are then determined. Choice of the form determines the least error that can be obtained in the regression process. Power laws are often used, though many other forms appear in the literature. Since digital computers are commonly used in heat rate calculations, it does not appear to be advantageous to have a simpler form of a correlation, as long as it is not unreasonably complex. In fact, to a certain extent accuracy in predictions is a more desirable goal than just simplicity of the correlation.

Symbolic regression is a procedure to find the form of the best-fitting correlation as well as the constants in it, and genetic programming offers a way to carry it out. The major virtue of the method is that no initial assumption of the functional form is needed. Since extremely large correlations may otherwise be generated, we have added a penalty function to the usual fitness function to prevent this from happening. We have demonstrated the application of this method using published data from two different experiments, and the resulting correlations, though slightly more complex than those generated from the traditional approach, have smaller predictive errors.

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References

[1] A.J. Pacheco-Vega, Simulation of Compact Heat Exchangers Using Global Regression and Soft Computing, Ph.D. Dissertation, University of Notre Dame, Notre Dame, IN, April 2002.
[2] A. Pacheco-Vega, G. Díaz, M. Sen, K.T. Yang, R.L. McClain, Heat rate predictions in humid air–water heat exchangers using correlations and neural networks, ASME J. Heat Transfer 123 (2) (2001) 348–354.
[3] A. Pacheco-Vega, M. Sen, K.T. Yang, Simultaneous determination of in- and over-tube heat transfer correlations in heat exchangers by global regression, Int. J. Heat Mass Transfer 46 (6) (2003) 1029–1040.
[4] S.W. Churchill, The art of correlation, Ind. Eng. Chem. Res. 39 (6) (2000) 1850–1877.
[5] F.P. Incropera, D.P. DeWitt, Fundamentals of Heat and Mass Transfer, John Wiley & Sons, New York, NY, 2002.
[6] A. Pacheco-Vega, M. Sen, K.T. Yang, R.L. McClain, Genetic-algorithm-based-predictions of fin-tube heat exchanger performance, in: J.S. Lee (Ed.), Proceedings of the Eleventh International Heat Transfer Conference, 6, Taylor & Francis, New York, NY, 1998, pp. 137–142.
[7] J.H. Holland, Adaptation in Natural and Artificial Systems, University of Michigan Press, Ann Arbor, MI, 1975.
[8] D.E. Goldberg, Genetic Algorithms in Search, Optimization and Machine Learning, Addison-Wesley, Reading, MA, 1989.
[9] J.R. Koza, Genetic Programming Paradigm, On the Programming of Computers by Means of Natural Selection, MIT-Press, Cambridge, MA, 1992.

[10] D.-G. Lee, H.-G. Kim, W.-P. Baek, S.H. Chang, Critical heat flux prediction using genetic programming for water flow in vertical round tubes, Int. Commun. Heat Mass Transfer 24 (7) (1997) 919–929.

[11] A. Pacheco-Vega, W. Cai, M. Sen, K.T. Yang, Heat transfer correlations in an air–water fin-tube compact heat exchanger by symbolic regression, in: Proceedings of the 2003 ASME International Mechanical Engineering Congress and Exposition, Washington, DC, November 2003, IMECE2003/HTD-41977.

[12] S.-H. Chen, C.-H. Yeh, Toward a computable approach to the efficient market hypothesis: an application of genetic programming, J. Econ. Dyn. Control 21 (1996) 1043–1063.

[13] J.F. Miller, D. Job, V.K. Vassilev, Principles in evolutionary design of digital circuits—part 1, Gen. Program. Evolv. Mach. 1 (2000) 7–35.

[14] K. Uesaka, M. Kawamata, Synthesis of low-sensitivity second-order digital filters using genetic programming with automatically defined functions, IEEE Signal Process. Lett. 7 (4) (2000) 83–85.

[15] V. Arkov, C. Evans, P.J. Fleming, D.C. Hill, J.P. Norton, I. Pratt, D. Rees, K. Rodriguez-Vazquez, System identification strategies applied to aircraft gas turbine engines, Ann. Rev. Control 24 (1) (2000) 67–81.

[16] S. Sette, L. Boullart, Genetic programming: principles and applications, Eng. Appl. Artificial Intell. 14 (1) (2001) 727–736.

[17] B. McKay, M. Willis, G. Barton, Steady-state modelling of chemical process systems using genetic programming, Comput. Chem. Eng. 21 (9) (1997) 981–996.

[18] D.E. Goldberg, K. Deb, A comparison of selection schemes used in genetic algorithms, in: G.J.E. Rawlins (Ed.), Foundations of Genetic Algorithms, Morgan Kaufmann, 1991, pp. 69–93.

[19] J.A. Nelder, R. Mead, A simplex method for function minimization, Comput. J. 7 (1965) 308–313.

[20] M. AbuMadi, R.A. Johns, M.R. Heikal, Performance characteristics correlation for round tube and plate finned heat exchangers, Int. J. Refrig. 21 (7) (1998) 507–517.

[21] N.H. Kim, B. Youn, R.L. Webb, Air-side heat transfer and friction correlations for plain fin-and-tube heat exchangers with staggered tube arrangements, ASME J. Heat Transfer 121 (3) (1999) 662–667.

[22] C.C. Wang, K.-U. Chi, C.-J. Chang, Heat transfer and friction characteristics of plain fin-and-tube heat exchangers, part II: Correlation, Int. J. Heat Mass Transfer 43 (15) (2000) 2693–2700.

[23] F.C. McQuiston, Correlation of heat, mass and momentum transport coefficients for plate-fin-tube heat transfer surfaces with staggered tubes, ASHRAE Trans. 84 (1) (1978) 294–309.

[24] H.F. Khartabil, R.N. Christensen, An improved scheme for determining heat transfer correlations from heat exchanger regression models with three unknowns, Exp. Thermal Fluid Sci. 5 (6) (1992) 808–819.

[25] F.C. McQuiston, Heat, mass and momentum transfer data for five plate-fin-tube heat transfer surfaces, ASHRAE Trans. 84 (1) (1978) 266–293.

[26] E.N. Sieder, G.E. Tate, Heat transfer and pressure drop of liquids in tubes, Ind. Eng. Chem. 28 (12) (1936) 1429–1435.

[27] O. Levenspiel, N.J. Weinstein, J.C.R. Li, A numerical solution to dimensional analysis, Ind. Eng. Chem. 48 (2) (1956) 324–326.